



Doubly charged vector tetraquark $Z_V^{++} = [cu][\bar{s}\bar{d}]$

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ABSTRACT

We explore properties of the doubly charged vector tetraquark $Z_V^{++} = [cu][\bar{s}\bar{d}]$ built of four quarks of different flavors using the QCD sum rule methods. The mass and current coupling of Z_V^{++} are computed in the framework of the QCD two-point sum rule approach by taking into account quark, gluon and mixed vacuum condensates up to dimension 10. The full width of this tetraquark is saturated by S -wave $Z_V^{++} \rightarrow \pi^+ D_{s1}(2460)^+$, $\rho^+ D_{s0}^*(2317)^+$, and P -wave $Z_V^{++} \rightarrow \pi^+ D_s^+$, $K^+ D^+$ decays. Strong couplings required to find partial widths of aforementioned decays are calculated in the context of the QCD light-cone sum rule method and a soft-meson approximation. Our predictions for the mass $m = (3515 \pm 125)$ MeV and full width $\Gamma_{\text{full}} = 156_{-30}^{+56}$ MeV of Z_V^{++} are useful to search for this exotic meson in various processes. Recently, the LHCb collaboration discovered neutral states $X_{0(1)}(2900)$ as resonance-like peaks in $D^- K^+$ invariant mass distribution in the decay $B^+ \rightarrow D^+ D^- K^+$. We argue that mass distribution of $D^+ K^+$ mesons in the same B decay can be used to observe the doubly charged scalar Z_S^{++} and vector Z_V^{++} tetraquarks.

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1. Introduction

Hadrons with unusual quantum numbers and/or multi quark-gluon contents attracted interest of researches starting from first days of the quark-parton model and Quantum Chromodynamics (QCD). It is known that conventional hadrons are composed of quark-antiquark pairs $q\bar{q}$, or made of three valence quarks qqq' . Masses and quantum parameters of ordinary mesons and baryons agree with predictions obtained in this scheme and can be calculated using standard methods of high energy physics. But fundamental principles of QCD do not forbid existence of particles built of four, five, etc. quarks, containing valence gluons or composed of exclusively valence gluons. Such hadrons may be produced in decays of B meson, in the electron-positron and proton-antiproton annihilations e^+e^- and $\bar{p}p$, in the double charmonium production processes, in the two-photon fusion and pp collisions.

The concept of multi-quark hadrons was applied by R. Jaffe to explain a mass hierarchy inside the lowest scalar multiplet [1]. In accordance with this assumption, mesons from the light scalar nonet can be interpreted as four-quark states $q^2\bar{q}^2$, which explain experimentally observed features of these particles. Another in-

teresting result about multi-quark hadrons was obtained also by R. Jaffe [2]. Thus, he considered six-quark states composed of only light quarks, and using the MIT quark-bag model calculated parameters of flavor-singlet and neutral bound state $uudds$ (H-dibaryon) with isospin $I = 0$ and spin-parity $J^P = 0^+$. If exist, this double-strange structure would be stable against strong decays, and have mean lifetime $\tau \approx 10^{-10}$ s, which is considerably longer than that of conventional mesons.

Properties of exotic hadrons were investigated in the context of QCD-inspired nonperturbative methods already at eighties of the last century. One of such effective methods is the QCD sum rule approach [3,4], which was employed to perform quantitative analyses and calculate masses and other parameters of the glueballs, hybrid $q\bar{q}g$ and four-quark mesons (tetraquarks) [3,5–10]. Unfortunately, achievements obtained at early stages of theoretical studies were not accompanied by reliable experimental measurements connected mainly with difficulties in detecting heavy resonances. As a result, existence of exotic hadrons was not then certainly established.

This situation changed starting from the Belle's discovery of the charmoniumlike resonance $X(3872)$ [11], which was confirmed later by other collaborations [12–14]. During followed years different experimental groups collected information about masses, widths, and quantum numbers of numerous heavy resonances,

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which may be considered as four-quark exotic mesons. There were attempts to describe new charmoniumlike states as excitations of the ordinary charmonium $c\bar{c}$: It is worth noting that some of them really allows such interpretation. But the bulk of available experimental data cannot be included into this scheme. These resonances may be considered in a tetraquark model, which treats them either as two-meson molecules or diquark-antidiquark states.

Observation of the charged resonances $Z^\pm(4430)$ and $Z_c^\pm(3900)$ had important impact on the physics of multiquark hadrons, because they could not be interpreted as neutral charmonia and became real candidates to tetraquarks [15,16]. Exotic mesons containing four quarks of different flavors also differ from charmoniumlike states and are promising candidates to genuine four-quark mesons. Analyses of such structures were inspired by information of the $D0$ collaboration about evidence for the state $X(5568)$ composed presumably of four different quarks [17]. Later, other collaborations could not confirm existence of this state [18,19], but knowledge gained due to theoretical studies of $X(5568)$, methods and schemes elaborated during this process played an important role in our understanding of exotic mesons. The resonances which are candidates to four-quark exotic mesons now form the wide family of XYZ states. Detailed information on XYZ particles including a history of the problem, as well as experimental results and theoretical achievements of last years are collected in numerous interesting review articles [20–30].

Recently the LHCb collaboration announced about new resonance-like structures $X_{0(1)}(2900)$ observed in the process $B^+ \rightarrow D^+D^-K^+$ [31,32]. They were seen in the D^-K^+ mass distribution, and can be considered as first strong evidence for exotic mesons composed of four quarks of different flavors. Indeed, from decay modes of these states, it is clear that they contain valence quarks $ud\bar{c}\bar{s}$. But one should bear in mind that $X_{0(1)}(2900)$ may have alternative origin, and appear due to rescattering diagrams $\chi_{c1}D^{*-}K^{*+}$ and $D_{s1}\bar{D}_1^0K^0$ in the decay $B^+ \rightarrow D^+D^-K^+$ [33].

The LHCb determined masses and spin-parities of these resonances, that were used in various models to explain internal organizations of $X_{0(1)}(2900)$. As usual, they were interpreted as hadronic molecules, diquark-antidiquark states, and rescattering effects (see, Refs. [34,35] and references therein). In our articles [34,35], we treated $X_0(2900)$ and $X_1(2900)$ as a scalar hadronic molecule $\bar{D}^{*0}K^{*0}$ and diquark-antidiquark vector states $[ud][\bar{c}\bar{s}]$, respectively. Predictions for their masses and widths extracted from sum rule analyses seem confirm assumptions made on their structures.

The resonances $X_{0(1)}(2900)$ emerge in intermediate phase of the decay chain $B^+ \rightarrow D^+X \rightarrow D^+D^-K^+$, and are neutral states. These processes occur due to color-favored and color-suppressed transformations of the B^+ meson [36]. But weak decays of B^+ with the same topologies may trigger also processes $B^+ \rightarrow D^-Z^{++} \rightarrow D^-D^+K^+$, where Z^{++} is a doubly charged exotic meson with quark content $cu\bar{s}\bar{d}$. In our view, experimental data collected by LHCb about weak decays of B meson may be used to uncover doubly charged tetraquarks $Z^{++} = [cu][\bar{s}\bar{d}]$ with different spin-parities. In fact, the scalar and vector particles Z_S^{++} and Z_V^{++} may appear as resonances in the D^+K^+ mass distribution, and provide valuable information on new exotic mesons.

Let us note that doubly charged diquark-antidiquarks already attracted interests of physicists, and some of them was studied in a rather detailed form. Thus, spectroscopic parameters and strong decays of pseudoscalar tetraquarks $cc\bar{s}\bar{s}$ and $cc\bar{d}\bar{s}$ were calculated in Ref. [37]. Doubly charged scalar, pseudoscalar and axial-vector states $[sd][\bar{u}\bar{c}]$ were considered in our article [38]. The tetraquarks Z^{++} are positively charged counterparts of states $[sd][\bar{u}\bar{c}]$ and have the same masses and decay widths. Therefore, one can safely use their parameters to study exotic mesons Z^{++} .

For completeness and following analysis, we provide masses and widths of various tetraquarks Z^{++} using, for these purposes, our results from Ref. [38]

$$\begin{aligned} m_{Z_S} &= 2628_{-153}^{+166} \text{ MeV}, \quad \Gamma_{Z_S} = (66.89 \pm 15.11) \text{ MeV}, \\ m_{Z_{PS}} &= 2719_{-156}^{+144} \text{ MeV}, \quad \Gamma_{Z_{PS}} = (38.1 \pm 7.1) \text{ MeV}, \\ m_{Z_{AV}} &= 2826_{-157}^{+134} \text{ MeV}, \quad \Gamma_{Z_{AV}} = (47.3 \pm 11.1) \text{ MeV}. \end{aligned} \quad (1)$$

Here, subscripts Z_S , Z_{PS} and Z_{AV} refer to the scalar, pseudoscalar and axial-vector Z^{++} , respectively.

As is seen, there are not predictions for parameters of the vector tetraquark Z_V^{++} , but its mass should be around or larger than $m_{Z_{AV}}$. Using this preliminary estimate for the mass of Z_V^{++} and m_{Z_S} from Eq. (1), we see that decays of Z_S^{++} and Z_V^{++} to ordinary mesons $D_s^+\pi^+$ and D^+K^+ are kinematically allowed processes, i.e., the tetraquarks Z_S^{++} and Z_V^{++} may be seen in D^+K^+ mass distribution in the decay $B^+ \rightarrow D^-D^+K^+$.

In the present article, we compute spectroscopic parameters and total width of the vector tetraquark Z_V^{++} using various versions of the QCD sum rule method. Our interest to this particle is twofold: First, it is a fully open flavor tetraquark, and additionally bears two units of electric charge. By theoretical exploration of Z_V^{++} , we will complete list of such states (1). Second reason is that, there are opportunities to fix the tetraquarks Z_S^{++} and Z_V^{++} using existing or future LHCb data.

We evaluate the mass and current coupling of Z_V^{++} by employing the QCD two-point sum rule approach [3,4]. In our analysis, we take into account contributions of various vacuum condensates up to dimension 10. Prediction for the mass of Z_V^{++} , as well as its quantum numbers $J^P = 1^-$ permit us to classify kinematically allowed decay modes of this tetraquark. The mass and coupling of Z_V^{++} are also input parameters necessary to calculate partial widths of the decays $Z_V^{++} \rightarrow D_{s1}(2460)^+\pi^+$, $D_{s0}^*(2317)^+\rho^+$, D^+K^+ , and $D_s^+\pi^+$. To this end, we explore vertices of Z_V^{++} with ordinary mesons (for example, $Z_V^{++}D^+K^+$), and find corresponding strong couplings. Relevant investigations are carried out using the QCD light-cone sum rule (LCSR) method, which is one of effective nonperturbative tools to study conventional hadrons [39]. In the case of tetraquark-ordinary meson vertices the standard methods of LCSR have to be applied in conjunction with a soft-meson approximation [40,41]. For analysis of the exotic mesons the light-cone sum rule method and soft approximation was suggested in Ref. [42], and used to investigate decays of various tetraquarks [30].

This paper is structured in the following manner: Section 2 is devoted to calculations of the mass and coupling of the vector tetraquark $Z_V^{++} = [cu][\bar{s}\bar{d}]$. In Section 3, we compute strong couplings in relevant tetraquark-meson vertices, and evaluate partial widths of Z_V^{++} decays. The full width of Z_V^{++} is found in this section as well. Section 4 is reserved for our conclusions and final notes.

2. The spectroscopic parameters of Z_V

The mass m and current coupling f are important parameters of the vector tetraquark $Z_V^{++} = [cu][\bar{s}\bar{d}]$ (in what follows, we omit superscripts denoting charges of the tetraquark and various mesons). We use the QCD two-point sum rule method to evaluate values of these parameters.

We begin calculations from analysis of the two-point correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle, \quad (2)$$

where $J_\mu(x)$ is the interpolating current for Z_V , and \mathcal{T} denotes the time-ordered product of two currents. The vector tetraquark

Z_V can be modeled using a scalar diquark $u_C\gamma_5c$ and vector antiquark $\bar{s}\gamma_\mu\gamma_5\bar{c}$. It is known that color-antitriplet diquarks and color-triplet antiquarks are most stable two-quark structures [20]. Therefore, we chose as building blocks of Z_V structures $\varepsilon^{abc}u_bC\gamma_5c_c$ and $\varepsilon^{amn}\bar{s}_m\gamma_\mu\gamma_5\bar{c}_n^T$, which belongs to $[\bar{3}_c]$ and $[3_c]$ representations of the color group $SU_c(3)$, respectively. Then, the colorless interpolating current $J_\mu(x)$ takes the form

$$J_\mu(x) = \varepsilon\tilde{\varepsilon}[u_b^T(x)C\gamma_5c_c(x)][\bar{s}_m(x)\gamma_\mu\gamma_5\bar{c}_n^T(x)], \quad (3)$$

where $\varepsilon\tilde{\varepsilon} = \varepsilon^{abc}\varepsilon^{amn}$, and a, b, c, m and n denote quark colors. In Eq. (3) $u(x), c(x), s(x)$ and $d(x)$ are the quark fields, and C stands for the charge-conjugation operator.

The sum rules for m and f can be derived by calculating $\Pi_{\mu\nu}(p)$ in terms of the physical parameters of Z_V , as well as in the operator product expansion (OPE) with certain accuracy. Having equated these two expressions, applied the Borel transformation to suppress contributions of higher resonances and continuum states, and subtracted these contributions using the quark-hadron duality assumption [3,4], it is possible to get required sum rules for the mass and coupling of the tetraquark Z_V .

Expression of $\Pi_{\mu\nu}(p)$ in terms of parameters of Z_V is obtained by saturating the correlation function $\Pi_{\mu\nu}(p)$ with a complete set of $J^P = 1^-$ states and performing in Eq. (2) integration over x

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{\langle 0|J_\mu|Z_V\rangle\langle Z_V|J_\nu^\dagger|0\rangle}{m^2 - p^2} + \dots, \quad (4)$$

where the dots stand for contributions of higher resonances and continuum states.

To proceed, it is convenient to introduce the matrix element

$$\langle 0|J_\mu|Z_V\rangle = f m \epsilon_\mu, \quad (5)$$

where ϵ_μ is the polarization vector of the tetraquark Z_V . Then $\Pi_{\mu\nu}^{\text{Phys}}(p)$ can be rewritten in the following form

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{m^2 f^2}{m^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} \right) + \dots. \quad (6)$$

The two-component Lorentz structure in the parentheses in Eq. (6) corresponds to a vector particle. The first component of this structure is proportional to $g_{\mu\nu}$ and does not contain effects of scalar particles: It is formed due to only vector states. Therefore, in our studies we utilize the structure $\sim g_{\mu\nu}$, and label corresponding invariant amplitude by $\Pi^{\text{Phys}}(p^2)$.

Expression in Eq. (4) is obtained in the zero-width single-pole approximation. In general, the correlation function $\Pi_{\mu\nu}(p)$ receives contributions also from two-meson reducible terms, because the current J_μ couples not only to four-quark structures, but also to two mesons with relevant quantum numbers [43,44]. Interactions with mesons lying below the mass of the Z_V generate a finite width of the tetraquark Z_V , and modify the quark propagator in Eq. (4)

$$\frac{1}{m^2 - p^2} \rightarrow \frac{1}{m^2 - p^2 - i\sqrt{p^2}\Gamma(p)}. \quad (7)$$

The two-meson contributions can be either subtracted from the sum rules or included into parameters of the pole term. In the case of the tetraquarks the second approach is preferable and was applied in articles [37,45,46]. These effects, properly taken into account in the sum rules, rescale the coupling f leaving stable the mass m of the tetraquark. Detailed analyses proved that two-meson contributions are small and can be neglected.

The QCD side of the sum rules $\Pi_{\mu\nu}^{\text{OPE}}(p)$ is found by inserting the interpolating current $J_\mu(x)$ into Eq. (2), and contracting relevant quark fields

$$\Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4x e^{ipx} \varepsilon\tilde{\varepsilon}\varepsilon'\tilde{\varepsilon}'\text{Tr} \left[\gamma_5 \tilde{S}_u^{bb'}(x) \gamma_5 S_c^{c'c}(x) \right] \times \text{Tr} \left[\gamma_\mu \gamma_5 \tilde{S}_d^{n'n}(-x) \gamma_5 \gamma_\nu S_s^{m'm}(-x) \right], \quad (8)$$

where

$$\tilde{S}_{c(q)}(x) = C S_{c(q)}^T(x) C, \quad (9)$$

with $S_c(x)$ and $S_{u(s,d)}(x)$ being the heavy c - and light $u(s, d)$ -quark propagators, respectively (for explicit expressions, see Ref. [30]). The invariant amplitude in $\Pi_{\mu\nu}^{\text{OPE}}(p)$ corresponding to the structure $g_{\mu\nu}$ in our following analysis will be denoted by $\Pi^{\text{OPE}}(p^2)$.

Calculations performed in accordance with a scheme briefly explained above yield

$$m^2 = \frac{\Pi'(M^2, s_0)}{\Pi(M^2, s_0)}, \quad (10)$$

and

$$f^2 = \frac{e^{m^2/M^2}}{m^2} \Pi(M^2, s_0), \quad (11)$$

which are sum rules for m and f , respectively. Here, $\Pi(M^2, s_0)$ is the Borel transformed and subtracted amplitude $\Pi^{\text{OPE}}(p^2)$, which depends on the Borel M^2 and continuum threshold s_0 parameters. In Eq. (10) $\Pi'(M^2, s_0)$ is the derivative of the amplitude over $d/d(-1/M^2)$.

It is clear that the amplitude $\Pi(M^2, s_0)$ is a key ingredient of the obtained sum rules. Calculations of this function give the following result

$$\Pi(M^2, s_0) = \int_{\mathcal{M}^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2} + \Pi(M^2), \quad (12)$$

where $\mathcal{M} = m_c + m_s$. Computations are carried out by taking into account vacuum condensates up to dimension 10. The amplitude $\Pi(M^2, s_0)$ has two components: First of them is expressed using the spectral density $\rho^{\text{OPE}}(s)$, which is computed as an imaginary part of $\Pi_{\mu\nu}^{\text{OPE}}(p)$. The Borel transformation of other terms is found directly from $\Pi_{\mu\nu}^{\text{OPE}}(p)$ and included into $\Pi(M^2)$. The first component in Eq. (12) contains a main part of the amplitude, whereas $\Pi(M^2)$ is formed from higher dimensional terms. Analytical expressions of $\rho^{\text{OPE}}(s)$ and $\Pi(M^2)$, are rather lengthy, therefore we do not write down them here.

The sum rules depend on vacuum condensates up to dimension 10. The basic condensates

$$\begin{aligned} \langle \bar{q}q \rangle &= -(0.24 \pm 0.01)^3 \text{ GeV}^3, \quad \langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle, \\ \langle \bar{q}q_s \sigma Gq \rangle &= m_0^2 \langle \bar{q}q \rangle, \quad \langle \bar{s}s_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle, \\ m_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2 \\ \langle \frac{\alpha_s G^2}{\pi} \rangle &= (0.012 \pm 0.004) \text{ GeV}^4, \\ \langle g_s^3 G^3 \rangle &= (0.57 \pm 0.29) \text{ GeV}^6, \end{aligned} \quad (13)$$

were estimated from analysis of numerous hadronic processes [3,4,47,48], and are known universal parameters. Higher dimensional condensates are factorized and expressed in terms of basic ones: we assume that factorization does not lead to large uncertainties. The masses of s and c quarks are also among input parameters, for which we use $m_s = 93_{-5}^{+11}$ MeV, and $m_c = 1.27 \pm 0.2$ GeV, respectively.

The Borel and continuum threshold parameters M^2 and s_0 are auxiliary quantities of computations and their choice is controlled by constraints imposed on the pole contribution (PC) and convergence of OPE, as well as by minimum sensitivity of the extracted quantities to the Borel parameter M^2 . Thus, the maximum allowed value of M^2 should be fixed to meet the restriction on PC

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}. \quad (14)$$

The lower limit of the working window for the Borel parameter is found from convergence of the operator product expansion, which is quantified by the ratio

$$R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)}. \quad (15)$$

Here $\Pi^{\text{DimN}}(M^2, s_0)$ denotes contribution of the last term (or a sum of last few terms) in the OPE. In the present work, at the maximum of M^2 we apply the constraint $\text{PC} > 0.2$, which is typical for the multiquark hadrons. To guarantee the convergence of the operator product expansion at minimum of M^2 , we use a requirement $R(M^2) \leq 0.01$.

Computations demonstrate that working regions for the parameters M^2 and s_0

$$M^2 \in [4.5, 6.5] \text{ GeV}^2, \quad s_0 \in [14, 16] \text{ GeV}^2, \quad (16)$$

satisfy aforementioned constraints. In fact, in these regions PC changes on average within limits

$$0.78 \leq \text{PC} \leq 0.28. \quad (17)$$

At the minimum $M^2 = 4.5 \text{ GeV}^2$, a contribution to $\Pi(M^2, s_0)$ arising from a sum of last three terms in OPE does not exceed 1% of the full result. In fact, at $M^2 = 4.5 \text{ GeV}^2$ the ratio $R(M^2)$ for $\text{DimN} = \text{Dim}(8 + 9 + 10)$ is equal to 0.007, which proves convergence of the sum rules.

Central values of the mass m and coupling f are evaluated at the middle point of regions (16), in other words, at $M^2 = 5.5 \text{ GeV}^2$ and $s_0 = 15 \text{ GeV}^2$. At this point the pole contribution is $\text{PC} \approx 0.56$, which ensures the ground-state nature of Z_V . Results for m and f read

$$\begin{aligned} m &= (3515 \pm 125) \text{ MeV}, \\ f &= (5.24 \pm 1.10) \times 10^{-3} \text{ GeV}^4. \end{aligned} \quad (18)$$

As it has been emphasized above, predictions extracted from sum rules should not depend on a choice of M^2 . In real analysis, however, there is a residual dependence on this parameter. In Fig. 1, we plot the mass of the tetraquark Z_V for wide range of M^2 and s_0 . It is seen, that only the region between two vertical lines in the left panel can be considered as a relatively stable plateau, where parameters of Z_V are evaluated. The current coupling f of the tetraquark Z_V is depicted in Fig. 2 as functions of M^2 and s_0 . In the case under discussion, one observes that f is almost stable in the explored range of the Borel parameter (16).

The second source of theoretical errors is connected with a choice of the continuum threshold parameter s_0 . Its working window should also satisfy limits stemming from dominance of PC and convergence of OPE. Despite M^2 , s_0 bears physical information about first excitation of the tetraquark Z_V . The self-consistent analysis implies that $\sqrt{s_0}$ has to be smaller than mass of such state. There are only a few samples when two observed resonances were assumed to be ground and radially excited states of the same tetraquark. The resonances $Z_c(3900)$ and $Z_c(4430)$ may form one of such pairs [49]. The difference between masses of $Z_c(3900)$ and

$Z_c(4430)$ is equal to $\approx 530 \text{ MeV}$, therefore a mass gap $\sqrt{s_0} - m \approx (400 - 600) \text{ MeV}$ can be considered as a reasonable estimate for tetraquarks. Here, we get on average $\sqrt{s_0} - m \approx 400 \text{ MeV}$ which is in accord with this general analysis.

Effects connected with a choice of parameters M^2 and s_0 are two main sources of theoretical uncertainties in sum rule computations. In the case of the mass m these ambiguities equal to $\pm 3.6\%$, whereas for the coupling f they are $\pm 21\%$ of the full result. Theoretical uncertainties for f are larger than for the mass, nevertheless, they do not overshoot accepted limits.

It is interesting to analyze a gap between masses of axial-vector and vector tetraquarks with the same content. Our present calculations demonstrate that for tetraquarks $[cu][\bar{s}\bar{d}]$ the difference between masses of the axial-vector and vector particles Z_{AV} and Z_V is $m - m_{Z_{AV}} \approx 690 \text{ MeV}$. This result can be compared with similar predictions for other tetraquarks. Thus, resonances $Y(4660)$ and $X(4140)$ with spin-parities $J^{\text{PC}} = 1^{--}$ and 1^{++} , quark content $[cs][\bar{c}\bar{s}]$ and color structure $[\bar{\mathbf{3}}_c] \otimes [\mathbf{3}_c]$ were investigated in Refs. [50,51], respectively. An estimate for $m_Y - m_{X_1}$ extracted from these studies is approximately equal to 500 MeV. In other words, radially excited axial-vector and ground-state vector tetraquarks, in this special case, are close in mass. This fact maybe useful to explain numerous charged and neutral resonances from XYZ family in the mass range of 4 – 5 GeV.

Another issue to be addressed here, is a mass gap between vector tetraquarks Z_V and $X_1(2900)$, which amounts to $\approx 600 \text{ MeV}$ and is quite large. It has been noted in section 1, that X_1 can be modeled as a vector tetraquark $[ud][\bar{c}\bar{s}]$, and hence both Z_V and X_1 are composed of same quarks. But there are two reasons, which may lead to aforementioned mass effect. First of them is internal organizations of these states: The tetraquark Z_V is composed of a heavy diquark $[cu]$ and relatively heavy antidiquark $[\bar{s}\bar{d}]$, whereas X_1 is strongly heavy-light compound. The latter is more tightly connected structure and should be lighter than Z_V . Besides, diquarks in Z_V have fractional positive electric charges which generate repulsive forces between them. In the case of X_1 , between $[ud]$ and $[\bar{c}\bar{s}]$ exists attractive electromagnetic interaction. Whether these features of Z_V and X_1 are enough to explain a mass gap between them or there are other sources of this effect, requires additional studies.

3. Strong decays of the tetraquark Z_V

The sum of partial widths of Z_V tetraquark's different decay channels constitutes its full width. The result for m obtained in the previous section is necessary to fix kinematically allowed strong decay modes of Z_V . Decays to final states $D_{s1}(2460)\pi$ and $D_{s0}^*(2317)\rho$ (below, simply $D_{s1}\pi$ and $D_{s0}^*\rho$) are among allowed S -wave process for the tetraquark Z_V . The P -wave processes, which will be explored, are decays $Z_V \rightarrow DK$ and $Z_V \rightarrow D_s\pi$.

3.1. Processes $Z_V \rightarrow D_{s1}\pi$ and $Z_V \rightarrow D_{s0}^*\rho$

Here, we study the decays $Z_V \rightarrow D_{s1}\pi$ and $Z_V \rightarrow D_{s0}^*\rho$, and compute their partial widths. We provide details of calculations for the first process and write down only essential formulas and final results for the second decay. It should be noted that the mass $m = (3515 \pm 125) \text{ MeV}$ makes possible S -wave decays of Z_V to final states $DK_1(1270)$, $D_s b_1(1235)$ and $D_s a_1(1260)$ as well. But widths of these processes, as it will be explained below, are suppressed relative to aforementioned two decays due to kinematical factors. Therefore, we restrict ourselves by investigation of two dominant S -wave decays.

The width of the process $Z_V \rightarrow D_{s1}\pi$ can be found using the coupling g_1 that describes strong interactions of particles Z_V , D_{s1} , and π at the vertex $Z_V D_{s1}\pi$. In order to evaluate g_1 , we use the

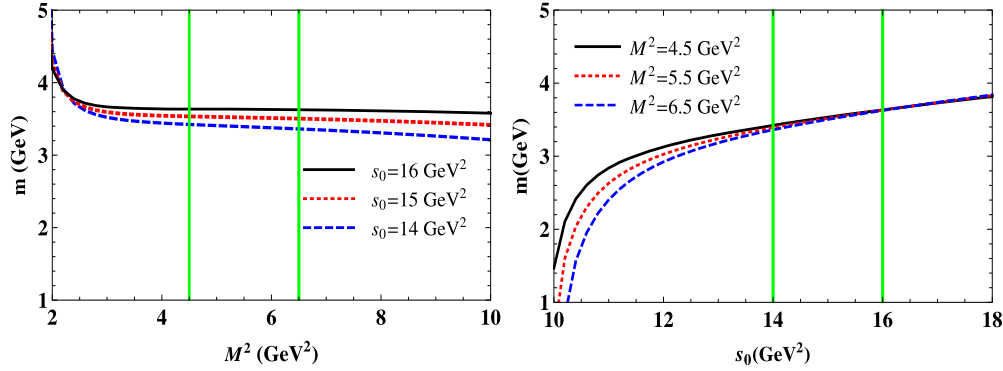


Fig. 1. Dependence of the Z_V tetraquark's mass m on the Borel parameter M^2 (left panel), and on the continuum threshold parameter s_0 (right panel). Vertical lines fix boundaries of working regions for M^2 and s_0 used in numerical computations.

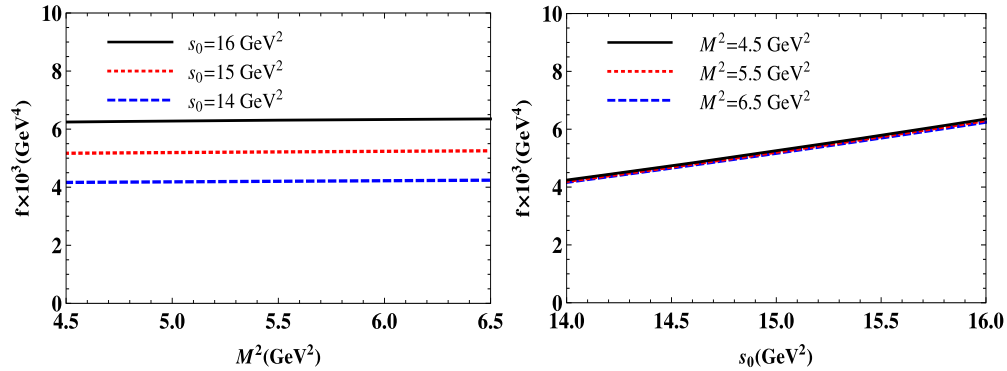


Fig. 2. The coupling f of the tetraquark Z_V as a function of M^2 (left panel), and s_0 (right panel). Variations of f are shown within working limits of the parameters M^2 and s_0 .

QCD light-cone sum rule method [39] and a soft-meson approximation [40,41].

Starting point in LCSR method is the correlation function

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{ipx} \langle \pi(q) | \mathcal{T} \{ J_\mu^{D_1}(x) J_\nu^\dagger(0) \} | 0 \rangle, \quad (19)$$

where by D_1 in the current $J_\mu^{D_1}$ we denote the meson D_{s1} . The current $J_\nu(0)$ in the correlation function $\Pi_{\mu\nu}(p, q)$ is introduced in Eq. (3). As interpolating current $J_\mu^{D_1}(x)$ for the axial-vector meson D_{s1} , we use the expression

$$J_\mu^{D_1}(x) = \bar{s}_l(x) i \gamma_5 \gamma_\mu c_l(x), \quad (20)$$

with l being the color index.

The function $\Pi_{\mu\nu}(p, q)$ has to be rewritten in terms of the physical parameters of the initial and final particles involved into the decay. By taking into account the ground states in the D_{s1} and Z_V channels, we get

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, q) &= \frac{\langle 0 | J_\mu^{D_1} | D_{s1}(p) \rangle \langle D_{s1}(p) \pi(q) | Z_V(p') \rangle}{p^2 - m_1^2} \\ &\times \frac{\langle Z_V(p') | J_\nu^\dagger | 0 \rangle}{p'^2 - m^2} + \dots, \end{aligned} \quad (21)$$

where p, q and $p' = p + q$ are the momenta of D_{s1}, π , and Z_V , respectively. In Eq. (21) m_1 is the mass of the meson D_{s1} , and the ellipses stand for contributions of higher resonances and continuum states in the D_{s1} and Z_V channels.

To get more detailed expression for $\Pi_{\mu\nu}^{\text{Phys}}(p, q)$, we utilize the matrix elements

$$\langle 0 | J_\mu^{D_1} | D_{s1} \rangle = f_1 m_1 \epsilon_\mu, \quad \langle Z_V(p') | J_\nu^\dagger | 0 \rangle = f m \epsilon_\nu^{*/}, \quad (22)$$

and model the vertex $\langle D_{s1}(p) K(q) | Z_V(p') \rangle$ in the following form

$$\begin{aligned} \langle D_{s1}(p) \pi(q) | Z_V(p') \rangle &= g_1 [(p \cdot p') (\epsilon^* \cdot \epsilon') \\ &\quad - (p \cdot \epsilon') (p' \cdot \epsilon^*)]. \end{aligned} \quad (23)$$

In Eqs. (22) and (23) f_1 and ϵ_μ are the decay constant and polarization vector of the meson D_{s1} , respectively: Polarization vector of the tetraquark Z_V in this section is denoted by ϵ'_ν . Using these matrix elements in Eq. (21), it is not difficult to find

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, q) &= g_1 \frac{m_1 f_1 m f}{(p^2 - m_1^2)(p'^2 - m^2)} \\ &\times \left(\frac{m^2 + m_1^2}{2} g_{\mu\nu} - p_\mu p'_\nu \right). \end{aligned} \quad (24)$$

The function $\Pi_{\mu\nu}^{\text{Phys}}(p, q)$ contains two structures proportional to $g_{\mu\nu}$ and $p_\mu p'_\nu$, respectively. These structures and relevant invariant amplitudes can be employed to extract the sum rule for the strong coupling g_1 . We choose to work with the term $\sim g_{\mu\nu}$ and corresponding invariant amplitude.

The QCD side of the sum rule can be obtained using explicit expressions of the correlation function $\Pi_{\mu\nu}(p, q)$ and interpolating currents. Having contracted quark and antiquark fields in the correlation function, we get

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(p, q) &= \int d^4x e^{ipx} \varepsilon \tilde{\varepsilon} \left[\gamma_5 \tilde{S}_c^{lc}(x) \gamma_\mu \gamma_5 \right. \\ &\quad \left. \times \tilde{S}_s^{ml}(-x) \gamma_\nu \gamma_5 \right]_{\alpha\beta} \langle \pi(q) | \bar{u}_\alpha^b(0) d_\beta^n(0) | 0 \rangle, \end{aligned} \quad (25)$$

where α and β are the spinor indexes.

The function $\Pi^{\text{OPE}}(p, q)$ contains local matrix elements of the quark operator $\bar{u}d$ sandwiched between the vacuum and pion. Simple transformations allow one to express $\langle \pi(q) | \bar{u}_\alpha^b(0) d_\beta^n(0) | 0 \rangle$ in terms of the pion's known local matrix elements. To this end, we expand $\bar{u}(0)d(0)$ by employing full set of Dirac matrices $\Gamma^J = \mathbf{1}, \gamma_5, \gamma_\mu, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}$, and project obtained operators onto the color-singlet states. These manipulations lead to replacement

$$\bar{u}_\alpha^b(0) d_\beta^n(0) \rightarrow \frac{1}{12} \delta^{bn} \Gamma_{\beta\alpha}^J [\bar{u}(0) \Gamma^J d(0)], \quad (26)$$

which should be fulfilled in $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$. The operators $\bar{u}(0) \Gamma^J d(0)$ placed between the vacuum and pion are matrix elements of the π meson.

The QCD expression of the correlation function (25) contains hard-scattering and soft parts. The hard-scattering part of $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$ is expressed in terms of quark propagators, whereas matrix elements of the pion form its soft component. For vertices built of conventional mesons correlation functions depend on non-local matrix elements of a final meson which are expressible in terms of its distribution amplitudes (DAs). In the case under analysis, due to four-quark nature of Z_V , $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$ contains only local matrix elements of the pion. As a result, integrals over DAs which are typical for LCSR method, reduce to overall normalization factors. In this case, the correlation function has to be found by means of the soft-meson approximation which implies computation of the hard-scattering part of $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$ in the limit $q \rightarrow 0$ [42]. It is worth to emphasize that soft-meson approximation is necessary to analyze tetraquark-meson-meson vertices: Strong couplings at vertices composed of two tetraquarks and a meson can be calculated by employing full version of the LCSR method [52].

It is evident, that soft approximation should be applied also to the phenomenological side of the sum rule. In the limit $q \rightarrow 0$ for the amplitude $\Pi^{\text{Phys}}(p^2)$, we get

$$\Pi^{\text{Phys}}(p^2) = g_1 \frac{f_1 m_1 f m}{(p^2 - \tilde{m}^2)^2} \tilde{m}^2 + \dots, \quad (27)$$

where $\tilde{m}^2 = (m^2 + m_1^2)/2$. The function $\Pi^{\text{Phys}}(p^2)$ depends on one variable $p^2 = p'^2$ and has the double pole at $p^2 = \tilde{m}^2$. The Borel transformation of $\Pi^{\text{Phys}}(p^2)$ is given by the formula

$$\Pi^{\text{Phys}}(M^2) = g_1 f_1 m_1 f m \tilde{m}^2 \frac{e^{-\tilde{m}^2/M^2}}{M^2} + \dots. \quad (28)$$

Besides ground-state contribution, the amplitude $\Pi^{\text{Phys}}(M^2)$ in the soft limit contains unsuppressed terms which survive even after Borel transformation. These contaminations should be removed from $\Pi^{\text{Phys}}(M^2)$ by applying the operator [40,41]

$$\mathcal{P}(M^2, \tilde{m}^2) = \left(1 - M^2 \frac{d}{dM^2}\right) M^2 e^{\tilde{m}^2/M^2}. \quad (29)$$

After this operation, remaining undesired terms in $\Pi^{\text{Phys}}(M^2)$ can be subtracted by usual manner in the context of quark-hadron duality assumption. It is clear, that we have to apply the operator $\mathcal{P}(M^2, \tilde{m}^2)$ to QCD side of the sum rule as well. Then, the sum rule for the strong coupling g_1 reads

$$g_1 = \frac{1}{f_1 m_1 f m \tilde{m}^2} \mathcal{P}(M^2, \tilde{m}^2) \Pi^{\text{OPE}}(M^2, s_0), \quad (30)$$

where $\Pi^{\text{OPE}}(M^2, s_0)$ is Borel transformed and subtracted invariant amplitude $\Pi^{\text{OPE}}(p^2)$ corresponding to the structure $g_{\mu\nu}$ in $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$.

Prescriptions to calculate the correlation function $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$ in the soft approximation were presented in Ref. [42], and in many

other publications, for this reason we do not concentrate here on details. Computations of $\Pi^{\text{OPE}}(M^2, s_0)$ performed in accordance with this scheme lead to the expression

$$\begin{aligned} \Pi^{\text{OPE}}(M^2, s_0) &= \frac{f_\pi \mu_\pi}{48\pi^2} \int_{\mathcal{M}^2}^{s_0} ds \frac{(m_c^2 - s)}{s^2} \\ &\times \left(m_c^4 + m_c^2 s + 6m_c m_s s - 2s^2 \right) e^{-s/M^2} \\ &+ \Pi_{\text{NP}}(M^2), \end{aligned} \quad (31)$$

where the first term is the perturbative contribution to $\Pi^{\text{OPE}}(M^2, s_0)$. The nonperturbative component $\Pi_{\text{NP}}(M^2)$ of the correlation function is calculated with dimension-9 accuracy, and has the following form

$$\begin{aligned} \Pi_{\text{NP}}(M^2) &= \frac{\langle \bar{s}s \rangle f_\pi \mu_\pi m_c (2M^2 + m_c m_s)}{12M^2} e^{-m_c^2/M^2} \\ &+ \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \frac{f_\pi \mu_\pi m_c}{144M^4} \int_0^1 \frac{dx}{X^3} \left[m_c^2 m_s X - 2m_s X M^2 \right. \\ &\left. + m_c^3 X + m_c X X M^2 \right] e^{m_c^2/[M^2 X]} + \frac{\langle \bar{s}g\sigma Gs \rangle f_\pi \mu_\pi}{72M^6} \\ &\times \left(m_s M^4 + m_s m_c^2 M^2 - 3m_c^3 M^2 - m_s m_c^4 \right) e^{-m_c^2/M^2} \\ &+ \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \langle \bar{s}s \rangle \frac{f_\pi \mu_\pi \pi^2 m_c (3M^2 - m_c^2)}{216M^8} \\ &\times \left(2M^2 + m_s m_c \right) e^{-m_c^2/M^2} + \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \langle \bar{s}g\sigma Gs \rangle \\ &\times \frac{f_\pi \mu_\pi \pi^2 m_c (m_c^5 m_s + 3M^2 m_c^4 - 7m_c^3 m_s M^2)}{1296M^{12}} \\ &- 18m_c^2 M^4 + 8m_c m_s M^4 + 18M^6 \Big) e^{-m_c^2/M^2}, \end{aligned} \quad (32)$$

where $X = x - 1$.

Contributions to $\Pi_{\text{NP}}(M^2)$ arise from the matrix element

$$\langle 0 | \bar{d} i \gamma_5 u | \pi \rangle = f_\pi \mu_\pi, \quad (33)$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2\langle \bar{q}q \rangle}{f_\pi^2}. \quad (34)$$

The last equality in Eq. (34) is the relation between the mass m_π and decay constant f_π of the pion, masses of quarks, and quark condensate $\langle \bar{q}q \rangle$, which stems from the partial conservation of the axial-vector current.

In numerical computations of $\Pi^{\text{OPE}}(M^2, s_0)$, we choose M^2 and s_0 within limits given by Eq. (16). Besides M^2 and s_0 , Eq. (30) contains various vacuum condensates and spectroscopic parameters of the final-state mesons D_{s1} and π . The masses and decay constants of the mesons D_{s1} and π , as well as other parameters used in numerical analyses are borrowed from Ref. [53] and presented in Table 1: For decay constants of the mesons D_{s0}^* and D_{s1} , we utilize the sum rule predictions from Refs. [48] and [54], respectively.

For the coupling g_1 , we get

$$g_1 = 0.36_{-0.06}^{+0.09} \text{ GeV}^{-1} \quad (35)$$

The partial width of the decay $Z_V \rightarrow D_{s1} \pi$ can be found by means of the formula

$$\Gamma_1 [Z_V \rightarrow D_{s1} \pi] = \frac{g_1^2 m_1^2 \lambda}{24\pi} \left(3 + \frac{2\lambda^2}{m_1^2} \right), \quad (36)$$

Table 1

Masses and decay constants of mesons used in numerical analyses.

Parameters	Values (in MeV units)
$m_1 = m[D_{s1}(2460)]$	2459.5 ± 0.6
$f_1 = f[D_{s1}(2460)]$	225 ± 25
$m_2 = m[D_{s0}^*(2317)]$	2317.8 ± 0.5
$f_2 = f[D_{s0}^*(2317)]$	202 ± 15
m_D	1869.65 ± 0.05
f_D	212.6 ± 0.7
m_{D_s}	1968.34 ± 0.07
f_{D_s}	249.9 ± 0.5
m_K	493.677 ± 0.0016
f_K	155.7 ± 0.3
m_π	139.57039 ± 0.00018
f_π	130.2 ± 1.2
m_ρ	775.26 ± 0.25
f_ρ	216 ± 3

where $\lambda \equiv \lambda(m, m_1, m_\pi)$ and

$$\lambda(a, b, c) = \frac{1}{2a} \left[a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2) \right]^{1/2}. \quad (37)$$

By employing this expression, it is not difficult to obtain

$$\Gamma_1[Z_V \rightarrow D_{s1}\pi] = 29.8_{-9.4}^{+17.6} \text{ MeV}. \quad (38)$$

The strong coupling and partial width of the second process $Z_V \rightarrow D_{s0}^*\rho$ can be found by the same manner. Here, one starts from the correlation function

$$\tilde{\Pi}_v(p, q) = i \int d^4x e^{ipx} \langle \rho(q) | \mathcal{T} \{ J^{D_0}(x) J_v^\dagger(0) \} | 0 \rangle, \quad (39)$$

where the interpolating current of the scalar meson D_{s0}^* is denoted by $J^{D_0}(x)$ and determined by the expression

$$J^{D_0}(x) = \bar{s}_l(x) c_l(x). \quad (40)$$

In our studies, we use the matrix element $\langle 0 | J^{D_0} | D_{s0}^* \rangle = f_2 m_2$, with m_2 and f_2 being the mass and decay constant of the D_{s0}^* . The vertex $Z_V D_{s0}^* \rho$ is modeled in the form

$$\langle D_{s0}^*(p) \rho(q) | Z_V(p') \rangle = g_2 \left[(q \cdot p') (\epsilon^* \cdot \epsilon') - (q \cdot \epsilon') (p' \cdot \epsilon^*) \right], \quad (41)$$

where ϵ_μ is the polarization vector of the ρ meson. Then, the phenomenological and QCD sides of the sum rule are given by expressions

$$\tilde{\Pi}_v^{\text{Phys}}(p, q) = g_2 \frac{m_2 f_2 m f}{(p^2 - m_2^2)(p'^2 - m^2)} \times \left(\frac{m_2^2 - m^2}{2} \epsilon_v^* + p' \cdot \epsilon^* q_v \right), \quad (42)$$

and

$$\tilde{\Pi}_v^{\text{OPE}}(p, q) = -i \int d^4x e^{ipx} \varepsilon \tilde{\varepsilon} \left[\gamma_5 \tilde{S}_c^l(x) \tilde{S}_s^{ml}(-x) \times \gamma_v \gamma_5 \right]_{\alpha\beta} \langle \rho(q) | \bar{u}_\alpha(0) d_\beta^m(0) | 0 \rangle, \quad (43)$$

respectively. We perform calculations using the structures $\sim \epsilon_v^*$ in $\tilde{\Pi}_v^{\text{Phys}}$ and $\tilde{\Pi}_v^{\text{OPE}}$. The relevant amplitude $\tilde{\Pi}^{\text{OPE}}(M^2, s_0)$ receives contributions from the matrix elements of the ρ meson

$$\begin{aligned} \langle 0 | \bar{d} \gamma_\mu u | \rho \rangle &= f_\rho m_\rho \epsilon_\mu, \\ \langle 0 | \bar{d} g \tilde{G}_{\mu\nu} \gamma_\nu \gamma_5 u | \rho \rangle &= f_\rho m_\rho^3 \zeta_4 \epsilon_\mu, \end{aligned} \quad (44)$$

where m_ρ and f_ρ are the mass and decay constant of the ρ meson, and $\tilde{G}_{\mu\nu}$ is the dual gluon field tensor $\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} / 2$. The second equality in Eq. (44) is the matrix element of the twist-4 operator [55]. The parameter $\zeta_{4\rho}$ was evaluated in the context of QCD sum rule approach at the renormalization scale $\mu = 1 \text{ GeV}$ in Ref. [56] and is equal to $\zeta_{4\rho} = 0.07 \pm 0.03$.

The correlation function $\tilde{\Pi}^{\text{OPE}}(M^2, s_0)$ has the form

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(M^2, s_0) &= \frac{f_\rho m_\rho}{16\pi^2} \int_{\mathcal{M}^2}^{s_0} \frac{ds(s - m_c^2)}{s} \\ &\times \left(m_c^2 + 2m_c m_s - s \right) e^{-s/M^2} + \frac{f_\rho m_\rho^3 \zeta_{4\rho}}{32\pi^2} \\ &\times \int_{\mathcal{M}^2}^{s_0} \frac{ds(m_c^4 - s^2)}{s^2} e^{-s/M^2} + \tilde{\Pi}_{\text{NP}}(M^2), \end{aligned} \quad (45)$$

where $\tilde{\Pi}_{\text{NP}}(M^2)$ is nonperturbative component of $\tilde{\Pi}^{\text{OPE}}$: We refrain from providing its explicit expression here.

Omitting further details, we write down results for the strong coupling g_2 and partial width of the process $Z_V \rightarrow D_{s0}^*\rho$

$$\begin{aligned} g_2 &= 0.61_{-0.12}^{+0.18} \text{ GeV}^{-1}, \\ \Gamma_2[Z_V \rightarrow D_{s0}^*\rho] &= 9.3_{-3.1}^{+6.5} \text{ MeV}. \end{aligned} \quad (46)$$

Returning to other S -wave decays listed above, we assume that couplings corresponding to vertices $Z_V DK_1(1270)$ etc., are the same order of g_1 and g_2 . Then widths of these decays are suppressed, because the factor $m_*^2 \lambda(3 + 2\lambda^2/m_*^2)$ (m_* is a mass of a final meson) is smaller for two final-state mesons of approximately equal mass than in the case of light and heavy mesons. We also take into account that the full width Γ_{full} of the tetraquark Z_V is formed mainly due to P -wave processes $Z_V \rightarrow DK$ and $Z_V \rightarrow D_s\pi$, and for this reason consider only two dominant S -wave decays.

3.2. Decays $Z_V \rightarrow DK$ and $Z_V \rightarrow D_s\pi$

In this subsection, we consider the P -wave decays $Z_V \rightarrow DK$ and $Z_V \rightarrow D_s\pi$ of the tetraquark Z_V . Treatments of these processes do not differ from analysis carried out above, differences being mainly in meson-tetraquark vertices and matrix elements of final-state mesons employed in calculations.

Let us concentrate on the decay $Z_V \rightarrow DK$. The correlation function to find a sum rule for the strong coupling G_1 of vertex $Z_V DK$ is given by the formula

$$\Pi_v(p, q) = i \int d^4x e^{ipx} \langle K(q) | \mathcal{T} \{ J^D(x) J_v^\dagger(0) \} | 0 \rangle, \quad (47)$$

where $J^D(x)$ is the interpolating current

$$J^D(x) = \bar{d}_l(x) i \gamma_5 c_l(x), \quad (48)$$

for the pseudoscalar meson D .

Then, the physical side of the sum rule has the form

$$\begin{aligned} \Pi_v^{\text{Phys}}(p, q) &= \frac{\langle 0 | J^D | D(p) \rangle \langle D(p) K(q) | Z_V(p') \rangle}{p^2 - m_D^2} \\ &\times \frac{\langle Z_V(p') | J_v^\dagger | 0 \rangle}{p'^2 - m^2} + \dots \end{aligned} \quad (49)$$

Using the matrix elements

$$\langle 0 | J^D | D \rangle = \frac{f_D m_D^2}{m_c}, \quad \langle D(p) K(q) | Z_V(p') \rangle = G_1 p \cdot \epsilon', \quad (50)$$

for Π_v^{Phys} , we find

$$\begin{aligned} \Pi_v^{\text{Phys}}(p, q) = & G_1 \frac{f_D m_D^2 f_m}{2m_c(p^2 - m_D^2)(p^2 - m^2)} \\ & \times \left[\left(-1 + \frac{m_D^2 - m_K^2}{m^2} \right) p_v + \left(1 + \frac{m_D^2 - m_K^2}{m^2} \right) q_v \right] \\ & + \dots \end{aligned} \quad (51)$$

In expressions above, m_D and f_D are the mass and decay constant of the D meson, respectively.

In terms of quark propagators the same correlation function $\Pi_v(p, q)$ is determined by the expression

$$\begin{aligned} \Pi_v^{\text{OPE}}(p, q) = & \int d^4x e^{ipx} \varepsilon \tilde{\varepsilon} \left[\gamma_5 \tilde{S}_c^{lc}(x) \gamma_5 \tilde{S}_d^{nl}(-x) \right. \\ & \left. \times \gamma_5 \gamma_\nu \right]_{\alpha\beta} \langle K(q) | \bar{u}_\alpha^b(0) s_\beta^m(0) | 0 \rangle. \end{aligned} \quad (52)$$

Operations necessary to derive the sum rule for the coupling G_1 have just been explained above, that is why we do not consider these questions. Let us note that the sum rule for G_1 has been obtained using the structures proportional p_v . The local matrix element of K meson which contributes to $\Pi_v^{\text{OPE}}(M^2, s_0)$ is

$$\langle 0 | \bar{d} i \gamma_5 u | \pi \rangle = \frac{f_K m_K^2}{m_s}, \quad (53)$$

where m_K and f_K are the mass and decay constant of the K meson.

The width of the process $Z_V \rightarrow DK$ can be found by means of the formula

$$\Gamma_3 [Z_V \rightarrow DK] = \frac{G_1^2 \lambda^3(m, m_D, m_K)}{24\pi m^2}. \quad (54)$$

In sum rule computations of the coupling G_1 the Borel and continuum threshold parameters are chosen as in Eq. (16). The spectroscopic parameters of the mesons D and K are collected in Table 1. Our predictions read

$$\begin{aligned} G_1 = & 4.3_{-0.7}^{+1.2}, \\ \Gamma_3 [Z_V \rightarrow DK] = & 34.6_{-10.9}^{+20.6} \text{ MeV}. \end{aligned} \quad (55)$$

For the second P -wave decay $Z_V \rightarrow D_s \pi$, we find

$$\begin{aligned} G_2 = & 6.6_{-1.1}^{+1.8}, \\ \Gamma_4 [Z_V \rightarrow D_s \pi] = & 81.8_{-25.8}^{+48.9} \text{ MeV}. \end{aligned} \quad (56)$$

Decay channels of the tetraquark Z_V considered in this section allow us to evaluate its full width

$$\Gamma_{\text{full}} = 156_{-30}^{+56} \text{ MeV}. \quad (57)$$

It is clear that Z_V can be classified as an exotic meson of wide width. Its main decay modes are processes $Z_V \rightarrow D_s \pi$ and $Z_V \rightarrow DK$ with branching ratios $\mathcal{BR}(Z_V \rightarrow D_s \pi) \approx 0.52$ and $\mathcal{BR}(Z_V \rightarrow DK) \approx 0.22$, respectively. Contribution of the decay $Z_V \rightarrow D_{s1} \pi$ is also considerable with estimate $\mathcal{BR}(Z_V \rightarrow D_{s1} \pi) \approx 0.19$.

4. Conclusions and final notes

In this article, we have studied the doubly charged vector tetraquark $Z_V^{++} = [cu][\bar{s}\bar{d}]$ and calculated its mass m and width Γ_{full} . The parameters of Z_V^{++} have been evaluated using the QCD two-point and light-cone sum rules. The doubly charged tetraquarks $[sd][\bar{u}\bar{c}]$ with spin-parities $J^P = 0^+, 0^-$ and 1^+ were investigated in our paper [38]. The exotic mesons Z^{++} are built

of four quarks of different flavors, and moreover bear two units of electric charge. These particles were not found till now, but due to unique features are interesting objects for both theoretical and experimental studies.

The LHCb collaboration recently observed structures $X_{0(1)}(2900)$ which may be interpreted as exotic mesons containing four different quarks [31,32]. The structures $X_{0(1)}$ were seen as resonance-like peaks in the mass distribution of $D^- K^+$ mesons. The master process to discover $X_{0(1)}$ was the weak decay of the B meson $B^+ \rightarrow D^+ D^- K^+$. It is remarkable that the process $B^+ \rightarrow D^+ D^- K^+$ and data collected during its exploration can be employed to observe another tetraquarks, namely states with quark content $cu\bar{s}\bar{d}$. In fact, analysis of the invariant mass distribution of mesons $D^+ K^+$ may lead to information about the scalar and vector tetraquarks Z_S^{++} and Z_V^{++} . Decays to final mesons $D^+ K^+$ are among favored channels of these particles. In fact, relevant branching ratios are equal to $\mathcal{BR}(Z_S \rightarrow DK) \approx 0.86$ and $\mathcal{BR}(Z_V \rightarrow DK) \approx 0.22$, respectively.

The decay of the B meson $B^+ \rightarrow D^- D_s^+ \pi^+$ is also useful to see states Z_S^{++} and Z_V^{++} , because processes $Z_S \rightarrow D_s \pi$ and $Z_V \rightarrow D_s \pi$ have considerable branching ratios $\mathcal{BR}(Z_S \rightarrow D_s \pi) \approx 0.12$ and $\mathcal{BR}(Z_V \rightarrow D_s \pi) \approx 0.52$. The decay $B^+ \rightarrow D^- D_s^+ \pi^+$ can be used to discover also tetraquarks $[ud][\bar{c}\bar{d}]$: These neutral states may be fixed in the invariant mass distribution of $D^- \pi^+$ mesons.

We have modeled Z_V^{++} as a four-quark exotic meson with diquark-antidiquark structure. Our analyses have proved that the interpolating current (3) used in the framework of the QCD sum rule method correctly describes the particle Z_V^{++} and leads to reliable results for its parameters. In fact, existence of the working windows for parameters M^2 and s_0 that satisfy standard requirements of the sum rules, and self-consistency of used $\sqrt{s_0}$ and extracted m argue in favor of this conclusion. It is known that a diquark-antidiquarks are tightly bound states, and may be favorite forms for doubly charged tetraquarks. But such four-quark systems may exist also as hadronic molecules. Thus, doubly charged molecular compounds with the quark content $\bar{Q}\bar{Q}qq$, where Q is c or b -quark were considered in Ref. [57]. In this article the authors used the heavy quark effective theory to derive interactions between heavy mesons, and coupled channel Schrodinger equations to find the bound and/or resonant states with various quantum numbers. It was demonstrated that, for example, D and D^* mesons can form doubly charged resonant state with $I(J^P) = 1(0^-)$. Similar analysis of a molecule counterpart of Z_V^{++} in the context of the QCD sum rule method implies usage of molecular-type interpolating current. Whether such current would lead to strong sum rule predictions or not requires detailed analysis, which however is beyond the scope of the present paper.

It is seen, that three-meson weak decays of B meson are sources of valuable information on four-quark exotic states. Data collected by various collaborations in running experiments can be utilized for these purposes. New decays of B meson can open wide prospects to study numerous four-quark states. Indeed, final-state mesons in such processes can be combined to form different pairs and their invariant mass distributions can be explored to detect resonance-type enhancements. In any case, additional experimental and theoretical studies are necessary to take advantage of emerging opportunities.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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